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13. ABSTRACT (Maximum 200 words)

Our work has been concerned with the utilization of new interpolation and operator theoretic methods for the study of problems in robust control. We have worked on possible nonlinear generalizations of H^∞ optimization theory and investigated corresponding intriguing questions in causality which arise in this context. In regard to our work on μ-synthesis and analysis, as well as the multivariable gain margin problem, we have developed a new type of interpolation method which is not norm-based. This is in contrast to the classical Nevanlinna-Pick theory which is used in H^∞ design. This line of research leads to spectral radius and structured interpolation extensions of the Nevanlinna-Pick framework. We have also discovered a new *lifting technique* for the robust stability analysis of systems with several kinds of structured perturbations. We worked in distributed (infinite dimensional) H^∞-control based on our *skew Toeplitz* methods. These have been applied to certain benchmark examples, e.g., delay systems that appear in aircraft control, and certain flexible beam problems. We have also been applying new curve evolution methods for problems in visual tracking.

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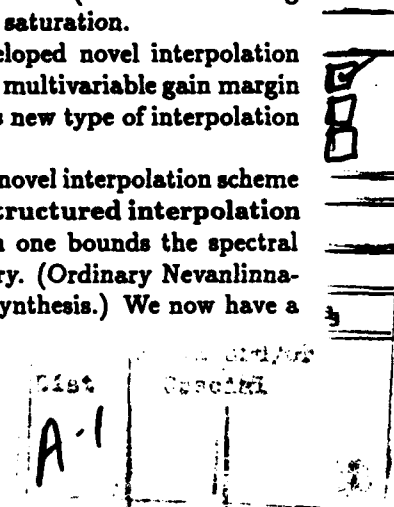April 1994

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1 Introduction

In our ARO work, we have carried out an extensive research program for the study of robust system control using methodologies from interpolation theory, dilation theory, and functional analysis. We have also become interested in image processing and computer vision, and their application to visual tracking problems.

First of all, we devoted a large portion of our research effort to nonlinear systems. This has led us to derive an *iterative commutant lifting theorem* which gives an explicit design procedure for nonlinear systems and captures the H^∞-control problem in the nonlinear framework [38], [39], [40]. In this area, we have defined a notion of *rationality* for nonlinear systems, and we have proven that the iterative commutant lifting procedure produces rational controllers (in this nonlinear sense) if we start from rational data. The procedure has already been applied to certain systems (connected with the National Aerospace Plane) with input saturations (in collaboration with colleagues at the Systems Research Center of Honeywell in Minneapolis). This framework has also led to new directions in introducing notions of causality [41] in commutant lifting theory, and moreover has led to the possible formulation of a global nonlinear commutant lifting theorem as a saddle-point result. We are particularly interested in the treatment (and understanding the control limitations) of "hard" nonlinearities such as dead-zone, backlash, and saturation.

During the research period supported by DAAL-03-91-G-0019, we also developed novel interpolation methods which are *not* norm-based. These have arisen out of our research into the multivariable gain margin problem as well as the more general *structured singular value*. We believe that this new type of interpolation theory should lead to several interesting directions in operator theory as well.

More precisely, our work on the multivariable gain margin problem, has led to a novel interpolation scheme which we call spectral Nevanlinna-Pick interpolation and the more general structured interpolation theory; see [14], [13], [15], and [17]. This involves matrix interpolation in which one bounds the spectral radius, and not the norm of the interpolants as in ordinary Nevanlinna-Pick theory. (Ordinary Nevanlinna-Pick is precisely the type of mathematical problem that arises in standard H^∞ synthesis.) We now have a

theoretical procedure for solving this spectral interpolation problem both in its pure matricial and tangential formulations.

Our solution involves a generalisation of the commutant lifting theorem, and a new object of interest in linear algebra and operator theory which we call the *T-spectral radius*. We are presently studying both the theoretical and practical ramifications of our solution, and we would like to develop software for the computation of the T-spectral radius. In our new research, we have been concentrating on the generalisation of these results to the structured singular value of Doyle and Safonov in order to try to develop an analytic μ-synthesis procedure [15]. Closely related to this work is a novel lifting technique which allows us to study the robust stability analysis of systems under various kinds of structured perturbations which we will consider below. (See also [9], [10], and [65].) This lifting technique allows one to interpret the upper bound for the structured singular value μ defined in terms of certain scalings as a structured singular value on a larger space. Moreover, we will discuss below a new approach for the approximation of Riemann mappings (conformal equivalences from simply connected sub-domains of the complex plane to the unit disc) for the application of solving the gain-phase margin problem [48].

We have continued our research on the utilisation operator theoretic methods in H^∞-optimisation theory using *skew Toeplitz operators* (see [12] for the precise definition), which seem ideally suited for studying H^∞ design problems, especially for distributed (i.e., infinite dimensional) systems. These methods are quite natural in the control context since they allow one to do design just using the input/output operators. This has led to an explicit solution of the four block problem for large class of multivariable distributed systems. (This is the most general H^∞-optimisation problem.) The procedure seems to be numerically robust as evidenced by its implementation at the Systems Research Center of Honeywell, and its application to a number of distributed plant models including a flexible beam [52], and an unstable delay system associated with the flight control of the X-29 [26]. A nice feature of this approach is that the complexity of the computations only depends on the the weighting matrices (modelling the disturbances) and not on the plant (which may be distributed). Since the weighting matrices are typically taken to be rational, this approach seems very efficient even in the finite dimensional case for plants with large state spaces. We have also been developing methods for the utilisation of an operator due to N. Young in order to simplify some of our computations in the *standard problem* for multivariable systems.

Next using the one-step dilation procedure, we have given a way of parametrising the suboptimal controllers for such generalised interpolation problems [34]. This has led to a scheme for designing suboptimal finite dimensional controllers for distributed systems in the one block setting [57]. We would like to continue this direction for the general standard problem as well as apply the techniques to a number of design problems. Some steps in this direction have already been taken in [26] and [52].

Closely related to the above work has been the use of H^∞ techniques to study sampled-data control. See [7] and the references therein. Here one can use a certain operator-theoretic lifting method to describe a complete solution to the analysis problem of verifying that a given controller constrains the L^2-induced norm of the sampled-data system to be less than some pre-specified level. (In fact, the lifting method is applicable to all norm based optimisation problems. In discrete-time such a technique was employed in [46] to study a number of issues in robust control.)

We will also discuss below our research in combined $H^\infty - H^2$ suboptimal controllers ([27] and [29]) as well as sketch an approach for combining other norms based on techniques from *interpolation on Banach spaces*, and the connection of this methodology to a new approach for model reduction which we intend to explore in our new ARO contract.

In a different direction, we have been using curve evolution theory (and especially invariant nonlinear diffusion equations), for the development of new algorithms in image processing and computer vision. Curve evolution theory has recently become a major topic of research, and indeed has appeared in such diverse fields as differential geometry, parabolic equations theory, numerical analysis, computer vision, the viscosity solutions of Hamilton-Jacobi equations, and image processing. See [49, 50, 51, 60, 61, 62, 63, 64] and the references therein.

In particular, evolution equations which are geometric non-linear versions of the classical heat equation have received much attention, since these equations have both theoretical and practical importance. These ideas have been used also in shape-from-shading, image smoothing and enhancement, motion planning, shape segmentation, optical flow, and a continuous implementation of mathematical morphology.

2 Nonlinear Robust Control

We have been pursuing several directions in order to derive nonlinear generalizations of the (linear) H^∞ theory in the weighted sensitivity minimization (one block), mixed sensitivity (two block), and even standard problem frameworks. One such direction is based on an iterative commutant lifting theorem [38], [39], [40], [32] which gives an explicit design procedure for nonlinear systems and captures the H^∞-control problem for a large class of nonlinear plants.

2.1 Causal Analytic Mappings

We consider analytic mappings on Hilbert space. For the precise definitions see [38], [39], [40], [30]. For simplicity, we will only consider SISO operators in what follows. The multivariable case works the same way [30]. We will find it convenient to employ the Fourier representation of elements of $H^2(D^n)$.

So we consider an analytic map ϕ with $\mathcal{G} = \mathcal{H} = H^2$ (the standard Hardy space on the disc D). Note that

$$H^2 \otimes \cdots \otimes H^2 = (H^2)^{\otimes n} \cong H^2(D^n)$$

where we map $1 \otimes \cdots \otimes z \otimes \cdots \otimes 1$ (z in the i-th place) to z_i, $i = 1, \cdots, n$. In the usual way, we say that ϕ *shift-invariant* (or *time-invariant*) if

$$\phi_n S^{\otimes n} = S\phi_n \quad \forall n \geq 1,$$

where $S : H^2 \to H^2$ denotes the canonical unilateral right shift. (Equivalently, this means that $S\phi = \phi \circ S$ on some open ball about the origin in which ϕ is defined.)

Now set

$$P^{(n)}_{(j)} := P_{(j)} \otimes \cdots \otimes P_{(j)} \ (n \text{ times}), \quad j \geq 1, \ n \geq 1,$$

where

$$P_{(j)} := I - S^j S^{*j}.$$

Then we say that ϕ is *causal* if

$$P_{(j)}\phi_n = P_{(j)}\phi_n P^{(n)}_{(j)}, \quad j \geq 1, \ n \geq 1.$$

For $\phi : H^2 \to H^2$ linear and time-invariant, it is easy to see that ϕ is causal. In the nonlinear setting however, time-invariance may not imply causality [40]. It is for this reason, that a causality constraint must be explicitly included for nonlinear H^∞ design.

2.2 Causal Optimization Problem

Because of the difficulties involving causality when one applies the classical commutant lifting theorem in the nonlinear framework, we will need to formulate a new linear causal optimization problem. Then we will indicate how to reduce the nonlinear generalization of the H^∞ sensitivity minimization problem to a series of such problems.

We let $S_{(n)}$ denote the unilateral shift on $H^2(D^n)$ given by multiplication by $(z_1 \cdots z_n)$. Since $H^2(D^n)$ will be fixed in the discussion we will let $S := S_{(n)}$. In what follows, U will denote the unilateral shift on H^2 given by multiplication by z, and $\Theta \in H^\infty$ will be an inner function. Finally $W : H^2(D^n) \to H^2$ will denote a causal, time-invariant bounded linear operator.

We can now state the *causal H^∞-optimization problem* (COP): Find

$$\sigma := \inf\{\|W - \Theta Q\| : Q : H^2(D^n) \to H^2, \ Q \text{ causal, time-invariant}\}. \tag{1}$$

Moreover, we want to compute an optimal, causal, time-invariant Q_{opt} such that

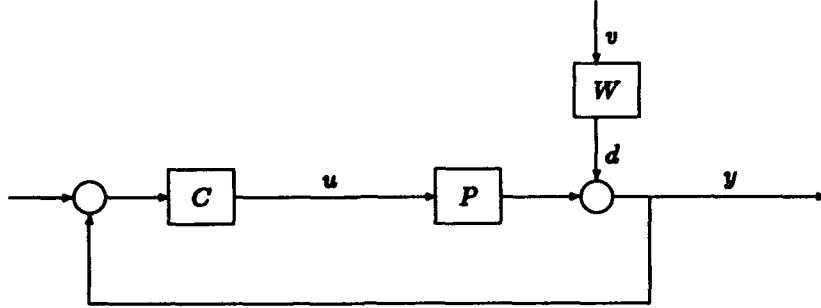$$\sigma = \|W - \Theta Q_{opt}\|. \tag{2}$$

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Figure 1: Sensitivity Minimisation Configuration

If we drop the causality constraint, the solution to problem (1) is provided by the classical commutant lifting theorem. With the causality constraint, the solution to (COP) is abstractly provided by a causal commutant lifting theorem [41], [33], [31].

Our constructions are based on a *reduction theorem* [30], which allows us to reduce the causal H^∞-optimisation problem to one of classical interpolation. Then in analogy with our previous work, we can base an causal iterative design procedure on this chain of ideas.

Here is how such a procedure would look. Let us call an analytic input/output operator $\phi : H^2 \to H^2$ *admissible* if is is causal, time-invariant, and $\phi(0) = 0$. Denote the set of admissible operators by C_a. In what follows below, we assume $P, W \in C_a$, and that W admits an admissible inverse.

Referring to Figure 1, we consider the problem of finding

$$\mu_\delta := \inf_C \sup_{\|v\| \leq \delta} \|[(I + P \circ C)^{-1} \circ W]v\|, \tag{3}$$

where we take the infimum over all stabilising controllers. (In what follows, we let $\| \ \|$ denote the 2-norm $\| \ \|_2$ on H^2 as well as the associated operator norm. The context will make the meaning clear.) Thus we are looking at a worst case disturbance attenuation problem where the energy of the signals v is required to be bounded by some pre-specified level δ. (In the linear case of course since everything scales, we can always without loss of generality take $\delta = 1$. For nonlinear systems, we must specify the energy bound *a priori*.) Then one sees that (3) is equivalent to the problem of finding the problem of finding

$$\mu_\delta = \inf_{q \in C_a} \sup_{\|v\| \leq \delta} \|(W - P \circ q)v\|. \tag{4}$$

The iterated causal commutant lifting procedure gives an approach for approximating a solution to such a problem. Briefly, the idea is that we write

$$
\begin{aligned}
W &= W_1 + W_2 + \cdots, \\
P &= P_1 + P_2 + \cdots, \\
q &= q_1 + q_2 + \cdots,
\end{aligned}
$$

where W_j, P_j, q_j are homogeneous polynomials of degree j. Notice that

$$\mu_\delta = \delta \inf_{q_1 \in H^\infty} \|W_1 - P_1 q_1\| + O(\delta^2), \tag{5}$$

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where the latter norm is the operator norm (i.e., H^∞ norm). From the classical commutant lifting theorem we can find an optimal (linear, causal, time-invariant) $q_{1,opt} \in H^\infty$ such that

$$\mu_\delta = \delta \|W_1 - P_1 q_{1,opt}\| + O(\delta^2). \tag{6}$$

Now the iterative procedure gives a way of finding higher order corrections to this linearisation. Let us illustrate this now with the second order correction. Indeed, having fixed the linear part $q_{1,opt}$ of q in (4), we note that

$$W(v) - P(q(v)) - (W_1 - P_1 q_{1,opt})(v) =$$

$$W_2(v) - P_2(q_{1,opt}(v)) - P_1 q_2(v) + \text{higher order terms.}$$

Regarding W_2, P_2, q_2 as linear operators on $H^2 \otimes H^2 \cong H^2(D^2, \mathbf{C})$ as above, we see that

$$\sup_{\|v\| \le \delta} \|(W - P \circ q)(v) - (W_1 - P_1 q_{1,opt})v\| \le \delta^2 \|\hat{W}_2 - P_1 q_2\| + O(\delta^3),$$

where the "weight" $\hat{W}_2$ is given by

$$\hat{W}_2 := W_2 - P_2(q_{1,opt} \otimes q_{1,opt}).$$

The point of the iterative causal commutant lifting procedure is to allow us to construct an optimal admissible $q_{2,opt}$, and so on.

In short, instead of simply designing a linear compensator for a linearisation of the given nonlinear system, this methodology allows one to explicitly take into account the higher order terms of the nonlinear plant, and therefore increase the ball of operation for the nonlinear controller. Moreover, if the linear part of the plant is rational, our iterative procedure may be reduced to a series of finite dimensional matrix computations. (See [30, 38] for discussions of rationality in the nonlinear framework.)

Finally, in a number of interesting cases, the above methodology can be extended to a nonlinear extension of the mixed sensitivity problem; see [25]. In fact, one can show that for linear weighting filters and admissible plant, the nonlinear mixed sensitivity problem may be reduced to a standard linear two block problem, followed by a one block nonlinear design, which may be solved using the above iterative methods.

Recently the above procedure was extended to the *general standard problem for nonlinear systems*; see [32]. We should note that we have concentrated on time-invariant systems in our above description. In fact, there is a much more general causal commutant lifting methodology [41, 33, 31] that can be extended in principle to *time-varying* systems. This is a very important problem area. Moreover, there are interesting problems concerning the equality of certain invariants which arise the the general causal commutant lifting framework [41] which we intend to consider in our upcoming ARO contract.

2.3 Saddle-Point Approach to Nonlinear Optimization

We have discussed above an iterative commutant lifting approach to nonlinear system design. The iterative commutant lifting technique is basically a local analytic method for nonlinear system synthesis. We have also been exploring a very different global approach based on an interpretation of the classical commutant lifting theorem as a saddle-point result [42]. This motivates us to formulate a nonlinear commutant lifting result in such a saddle-point, game-theoretic framework. A related approach to nonlinear design has already been independently employed in the novel and important papers of Ball-Helton [4], [5].

In our research, instead of considering general nonlinear systems we limit ourselves to the concrete (but certainly interesting case) of linear systems with input saturations. Such systems occur, of course, all the time in "nature." We should add that a similar approach should be valid for many of the hard, memoryless, noninvertible nonlinearities which appear in control.

Mathematically, the case of the saturation is very interesting since in a certain sense it is a nonlinear analogue of an inner (non-minimum phase) element, whose "spectrum" seems to be spread throughout the unit circle. Thus the problem of sensitivity minimisation for such elements (that is, a form of weighted inversion) is particularly challenging, and will be a key topic to be studied in our upcoming research program. What needs to be developed for this global approach to nonlinear H^∞ is a commutant lifting type result valid on convex spaces. We are doing this in our new ARO contract.

3 Structured Singular Values and μ-Synthesis

We would like now to discuss our work on the structured singular value and μ-synthesis using a novel approach which we developed called *structured interpolation theory*. The structured singular value was introduced into robust control by John Doyle and Michael Safonov ([22], [59]) to handle problems involving structured perturbations which includes both H^∞ and the multivariable gain margin as special cases.

Our starting point is the fact that the problem of internal stabilization of a given LTI multivariable plant can be reduced to one of interpolation. Thus the natural measure of robust performance in this framework is given by the minimisation of the structured singular values over all possible interpolants. Hence, one needs a structured Nevanlinna-Pick type result which will generalise both the classical Nevanlinna-Pick theorem (relative to the H^∞-norm), as well as the spectral Nevanlinna-Pick theorem (relative to the spectral radius). Such a structured Nevanlinna-Pick theorem can be deduced as a consequence of a general structured commutant lifting theorem as we will indicate below. This approach has been developed in our ARO sponsored work [13, 14, 11, 17, 8].

We will discuss some of these issues now, and explain how these results can serve as a basis of performing the μ-synthesis procedure in robust feedback control in a rigorous, analytical manner.

3.1 Structured Singular Values and Dilations

We would like to formally introduce the structured singular value now, and give some of its basic properties. We base this discussion on [15]. Instead of working over diagonal sets of matrices as in [22], we can more generally work over an arbitrary finite dimensional C^*-algebra.

Let $\mathcal{E}$ be a complex finite-dimensional Hilbert space, and $\Delta \subset \mathcal{L}(\mathcal{E})$ (the space of bounded linear operators on $\mathcal{E}$), a C^*-algebra. For $A \in \mathcal{L}(\mathcal{E})$, $A \neq 0$, we define the *structured singular value*

$$\mu_\Delta(A) := [\inf\{\|X\| : X \in \Delta, -1 \in \sigma(AX)\}]^{-1}.$$

Moreover, we set

$$\widehat{\mu}_\Delta(A) := \inf\{\|XAX^{-1}\| : X \in \Delta'\},$$

where Δ' is the commutator of Δ. Note that for $\Delta = \mathcal{L}(\mathcal{E})$, $\mu_\Delta(A) = \|A\|$, while for $\Delta = \mathbf{C}I_\mathcal{E}$, $\mu_\Delta(A) = \|A\|_{sp}$ (the spectral radius of A).

For certain diagonal algebras of matrices, it is argued in [22] and [59] that the structured singular value μ_Δ is the natural object of study in robust control. Unfortunately the structured singular value is difficult to compute, so in practice it is $\widehat{\mu}_\Delta$ which is actually used for control problems. It is therefore of interest to know when these two objects are equal.

In [22], Doyle has shown that in fact $\mu_\Delta = \widehat{\mu}_\Delta$ when the relevant diagonal algebra has three or fewer blocks. In [15], we give a very different proof of this fact based on the following result which we believe has independent interest. Define the operator $M_A \in \mathcal{L}(\mathcal{E})$ by $M_A := AX$. Notice that $\mathcal{L}(\mathcal{E})$ may be given a Hilbert space structure with respect to

$$\langle T_1, T_2 \rangle := Tr(T_2^* T_1),$$

where Tr denotes the trace. Define

$$\bar{\mu}_\Delta(A) := \mu_{\tilde{\Delta}}(M_A)$$

where

$$\tilde{\Delta} := \{M_X : X \in \Delta'\}'.$$

We now have (see [15] for the proof):

Theorem 1

$$\widehat{\mu}_\Delta(A) := \bar{\mu}_\Delta(A).$$

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Note that Theorem 1 implies that $\widehat{\mu}_\Delta$ can be regarded as a structured singular value on a bigger space. From the theorem and property (ii) above, we can immediately infer the key fact that $\widehat{\mu}_\Delta$ is continuous. This result is also strongly connected to some recent work on robust stability with respect to time-varying perturbations [65]. In fact, we are investigating the extension of this lifting technique to operators on infinite dimensional Hilbert spaces with applications to the robust stability analysis of systems under various kinds of structured perturbations [9]. In particular, we have considered the following two cases [9, 10]:

(i) The algebra of constant diagonal scales Δ', with the operator A taken to be analytic Toeplitz. (This is the case analysed in [65].) Note that by "analytic Toeplitz operator" A, we mean that A is given by an $n \times n$ matrix with entries H^∞ functions, which we regard as acting as a multiplication operator on $H^2(\mathbf{C}^n)$. Thus A defines a stable LTI system. We also want to consider general time-varying linear input-output operators with this constant diagonal algebra of scalings as well.

(ii) Again we take A to be analytic Toeplitz, but now we want to consider Δ' to be the algebra of diagonal analytic Toeplitz operators. This is the type of problem considered in μ-synthesis.

We should note that this work has also led to a new *relative Toeplitz-Hausdorff theorem*.

We will now outline a structured analogue of the commutant lifting theorem [15]. This will be applied to the structured version of classical matricial Nevanlinna-Pick interpolation below.

Set $T := S(m) \otimes I_\mathcal{E}$ where m is a finite Blaschke product, $S(m)$ is the compressed shift, and $\mathcal{E}$ is a finite dimensional complex Hilbert space. Fix $\Delta \subset \mathcal{L}(\mathcal{E})$, a C^*-algebra. Define

$$I_{H^2} \otimes \Delta := \{I_{H^2} \otimes X : X \in \Delta\}, \quad I_{H^2} \otimes \Delta' := \{I_{H^2} \otimes X : X \in \Delta'\}.$$

Notice that $\mathcal{H} := H^2(\mathcal{E}) \ominus m H^2(\mathcal{E})$ reduces both $I_{H^2} \otimes \Delta$ and $I_{H^2} \otimes \Delta'$. Now define for $A \in \{T\}'$ (the commutant of T),

$$\rho_T^\Delta(A) := \inf\{\|XAX^{-1}\| : X \text{ invertible}, \ X \in \{T\}', \ X \in (I_{H^2} \otimes \Delta|\mathcal{E})'\}.$$

Let U be the isometric dilation of T on $H^2(\mathcal{E})$ (so that U is defined by multiplication by z). For $B \in \{U\}'$, define

$$\rho_U^\Delta(B) := \inf\{\|YBY^{-1}\| : Y \text{ invertible}, \ Y \in \{U\}', \ Y \in (I_{H^2} \otimes \Delta)'\}.$$

We can now state the following result from [15]:

Theorem 2 (Structured Commutant Lifting Theorem) *Notation as above. Then for $A \in \{T\}'$,*

$$\rho_T^\Delta(A) = \inf\{\rho_U^\Delta(B) : B \text{ is a commuting dilation of } A\}.$$

Actually, in the finite dimensional case of interest to us, we can show that

$$\rho_T^\Delta(A) = \inf_B \sup_{z \in D} \widehat{\mu}_\Delta(B(z))$$

where $B \in \{U\}'$ is a commuting *rational* dilation of A. This leads to structured versions of the classical Nevanlinna-Pick theory both in its matricial and tangential forms [15, 11, 8]. The tangential form is precisely what is needed in making the μ-synthesis procedure rigorous.

3.2 Conformal Mappings

We would like to briefly discuss some work that we also did on the gain-phase margin problem that involves some novel approximation methods of Riemann mappings (conformal equivalences from a simply connected proper subset of C to the unit disc) due to Marshall-Morrow (see [20] and the references therein).

The successful solution of the gain and phase margin problems (see [67], [48], [23]) was based on the fact that one could explicitly construct a conformal equivalence from a simply connected subdomain of C (which was associated to the uncertainty model) to the disc. Nevanlinna-Pick interpolation then took care of the rest. In [48], we considered a combination of both the classical gain and phase margins in order to arrive at a better measure of robust stability which we called the *gain-phase margin*. Unfortunately, in this case, there is no explicitly computable conformal equivalence between the associated domain of uncertainty and the disc. The algorithm of Marshall-Morrow however gives a fast way of approximating the given equivalence, and has led to a reliable approximate solution of gain-phase margin in [20]. We are now applying this algorithm to the type of general robust synthesis question as considered by Sideris and Safonov in [66].

4 H^∞ Optimization of Distributed Systems

In [53], [54] we have considered the mixed sensitivity H^∞-optimisation problem for distributed plants with a finite number of unstable poles.

As in the standard operator approach to such problems, the computation of the optimal performance and corresponding optimal controller is reduced to a finite dimensional matrix problem. In the stable case, the size of the matrix only depends on the McMillan degree of the weighting filters. In the case of unstable plants, the size of the corresponding matrix depends on the number of right half plane poles of the plant as well. The dimension of this matrix can be computed *a priori*. The key mathematical fact that we use is that the skew Toeplitz operators which we obtain in the unstable case are finite rank perturbations of the classical skew Toeplitz operators obtained from compressions of rational functions.

In our ARO research, we have used these techniques to synthesise controllers for several types of flexible structures [52], and an unstable delay system derived from the flight control of the X-29 [26].

4.1 Two Block Problems With Unstable Plant

In this section, we will show that several two block H^∞-minimisation problems reduce to the computation of the norm of a certain skew Toeplitz operator, and indicate how this norm may be computed. We begin with some notation. The Hardy spaces H^2 and H^∞ are defined on the unit disc as above. We denote

$$RH^\infty := \{\text{rational functions in } H^\infty\}.$$

We consider the feedback configuration of Figure 2 with

$$P = \frac{G_n}{G_d}$$

and $G_n \in H^\infty$, $G_d \in RH^\infty$.

We assume that (i) $G_n = m_n G_{no}$, where $m_n \in H^\infty$ is inner (arbitrary) and $G_{no} \in H^\infty$ is outer, and (ii) G_n, G_d have no common zeros in the closed unit disc. We also write $G_d = m_d G_{do}$ where $m_d \in RH^\infty$ is inner and $G_{do} \in RH^\infty$ is outer. Under these assumptions there exist $X \in RH^\infty$ and $Y \in H^\infty$ such that

$$XG_n + YG_d = 1. \tag{7}$$

The set of all controllers which stabilise the plant can now be written in the form

$$C = \frac{X + QG_d}{Y - QG_n}$$

for some $Q \in H^\infty$. Now let $S := (1 + PC)^{-1}$ and note that

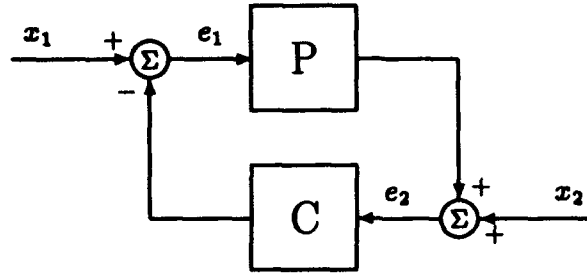$$S = 1 - XG_n - QG_nG_d. \tag{8}$$

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Figure 2: Standard Feedback Configuration

In [53], [54], we show that the computation of

$$\mu = \inf_{\text{stabilising } C} \left\| \begin{bmatrix} W_1 S \\ W_2(S-1) \end{bmatrix} \right\|$$

where W_1, $W_2 \in RH^\infty$ are given weighting functions with W_1^{-1}, $W_2^{-1} \in RH^\infty$ may be reduced to computing the norm of the operator

$$A := \begin{bmatrix} \mathbf{P}_{H(m_v)} \left(W_0(\mathbf{S}) - \hat{W}_0(\mathbf{S})m(\mathbf{S}) \right) \\ G_0(\mathbf{S}) \end{bmatrix}, \tag{9}$$

where $\mathbf{S} : H^2 \to H^2$ denotes the unilateral shift, $H(m_v) := H^2 \ominus m_v H^2$ and $\mathbf{P}_{H(m_v)}$ the orthogonal projection onto $H(m_v)$, for m, m_v inner functions associated to the plant and weighting filters, and where $W_0, \hat{W}_0, G_0$ are rational H^∞ functions computed from the plant and weighting filters. This reduction is true for plants with arbitrary outer parts.

In [53], [54], we develop an approach to computing the singular values and vectors of operators of the form (9). We remark here that it is easy to compute the essential norm (see [53], [54]) of the operator A, which will be denoted by $\|A\|_e$. Then in [54], the following result is proven:

Theorem 3 *Let n denote the maximum of the McMillan degrees of the weighting filters W_1 and W_2, and let ℓ denote the number of unstable poles of the plant P. Then the singular vectors and values of A which are $> \|A\|_e$ may be derived from an explicitly computable system of $3n + 2\ell$ linear equations (the "singular system").*

In [54], the singular system of equations is explicitly written down. Again the number of equations only depends on the McMillan degrees of the weighting filters, and the number of right half plane poles of the plant. The computation of the *maximal* singular value and the associated singular vectors of A, then allows us to find the optimal performance μ of our original control problem and the corresponding optimal compensator.

4.2 Mixed Norm Suboptimal Controllers

One of the main themes in modern control theory is the utilisation of norm based criteria to measure the optimal performance of a given control system. (See [23] for detailed discussions on this topic.) Two of the most important norms employed in modern control analysis and design are the H^2-norm and the H^∞-norm. Indeed, the H^2-norm is the basis of classical quadratic optimal control, while the H^∞-norm appears in modern robust synthesis (and implicitly in the more classical loop shaping methods [23]).

We have been investigating combining the advantages of both H^2 and H^∞ control in [29]. (See also [27] for extensive lists of references on various approaches to mixed norm control.) Our starting point is a nice

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result of Kaftal-Larson-Weiss [45] which guarantees the existence of an interpolant which simultaneously satisfies an H^∞ and H^2 suboptimality criterion. Using the theory described in [34] on suboptimal H^∞ interpolants, we can then give an explicit way of computing the combined H^∞-H^2 suboptimal interpolant in a given H^∞ problem.

More precisely, for $c \in L^\infty$, set $d_\infty(c) :=$ distance(c, H^∞), and $d_2(c) :=$ distance(c, H^2). (Note that all of our Hardy spaces are defined on the unit disc here.) Then for any $\delta > 1$, it is proven in [45] that there exists $\phi \in H^\infty$ such that

$$\|c - \phi\|_\infty \le \delta d_\infty(c), \quad \|c - \phi\|_2 \le \frac{\delta d_2(c)}{\sqrt{\delta^2 - 1}}. \tag{10}$$

In case, $m, w \in H^\infty$ with m inner, and $w = p/q$ rational, using skew Toeplitz theory [29] one may explicitly construct ϕ, i.e., if $c := \overline{m}w$, then we can find $\phi \in H^\infty$ satisfying

$$\|\overline{m}w - \phi\|_\infty \le \delta d_\infty(c), \quad \|\overline{m}w - \phi\|_2 \le \frac{\delta d_2(c)}{\sqrt{\delta^2 - 1}}. \tag{11}$$

Indeed, let $\rho := \delta d_\infty(c)$, and let $\hat{A}_\rho := \rho^2 q(T)^* q(T) - p(T)^* p(T)$ be a skew Toeplitz operator. Let g_δ denote the function in H^∞ defined by

$$g_\delta := \delta^2 d_\infty(c)^2 \hat{A}_\rho^{-1} q(T)^* (1 - m\overline{m(0)}). \tag{12}$$

Then the function

$$w - m\phi = B_\delta = \frac{p(T)g_\delta}{m\overline{m(0)} + q(T)g_\delta} \tag{13}$$

satisfies (11).

The skew Toeplitz approach is the only procedure of which we are aware to compute a ϕ satisfying (11) when m is irrational. Further, let $n := \max\{\text{degree } p, \text{degree } q\}$. Then, if m is Blaschke product of order k where n is small and k is large, the methodology just outlined provides a numerically efficient method for computing the function ϕ satisfying the Kaftal-Larson-Weiss constraints (11). However, if this is not the case and m is rational, then the state space methods in [27] to compute a ϕ satisfying (11) may be more efficient.

Since as it turns out that the combined H^∞-H^2 interpolant is a central solution, these interpolants are well-known to be numerically robust, and leads to an interesting class of combined H^∞-H^2 controllers for feedback systems. We are now actively investigating the properties of these controllers, as well as extending these results to find a computational procedure for the *full standard problem* setting for distributed MIMO systems.

Further, we have found a new method for constructing the *central solutions* as related to the general H^∞ standard problem in the multivariable case. We are now in the process of exploiting this technique for the explicit parametrization of the suboptimal controllers for multivariable distributed systems. This would give a powerful alternative method to the one-step extension techniques we have been previously using.

Finally, we are interested in combining other types of norms in control, for example, L^1 and H^∞. A possible method for doing this can be based on some new results on interpolation in Banach spaces due to Pisier [58]. In fact, this methodology can also have some nice applications to model reduction (relative to "best" Hankel norm approximations), which is certainly a direction which we intend to explore.

5   Image Processing and Computer Vision

We have been doing extensive research into image processing and computer vision, and the use of these methodologies in visual tracking. We are very interested in understanding how to use visual information in a feedback loop. In order to give the reader a flavor of this work, we will concentrate on a new multiresolution representation theory that we developed for planar shapes [60, 61]. In this work, we combine the three important theories of scale-spaces, affine invariant descriptors, and curve evolution, in order to define a new affine invariant scale-space for plane curves. This scale-space is obtained from the solution of a novel

curve flow, which admits affine invariant solutions. Our other work in vision covers a new theory of shape [49, 50, 51], as well as new methods for affine invariant image processing and analysis [62, 63, 64].

Multi-scale descriptions of signals have been studied for several years already. A possible formalism for this topic comes from the idea of multi-scale filtering which was introduced by Witkin [68], and developed in several different frameworks by a number of researchers in the past decade. The idea of scale-space filtering is very simple and can be formulated as follows: Given an initial signal $\Psi_0(\vec{X}) : \mathbf{R}^n \to \mathbf{R}^m$, the scale-space is obtained by filtering it with a kernel $\mathcal{K}(\vec{X}, t) : \mathbf{R}^n \to \mathbf{R}^m$, where $t \in \mathbf{R}^+$ represents the scale. In other words, the scale-space is given by $\Psi(\vec{X}, t)$ defined as

$$\Psi(\vec{X}, t) := \Omega_{\mathcal{K}(\vec{X},t)}[\Psi_0(\vec{X})], \tag{14}$$

where $\Omega_{\mathcal{K}(\cdot,t)}[\cdot]$ represents the action of the filter $\mathcal{K}(\cdot, t)$. Larger values of t correspond to images at coarser resolutions.

It is important to note that not every kernel can be used in defining a scale-space. Indeed, several conditions need to be imposed on the signal $\Psi(\vec{X}, t)$ (and therefore on the filtering operation (14)). One of the most important is that of *causality*, which states that no "information" is created when moving from fine to coarse scales.

A famous example of a kernel which satisfies the required conditions is the Gaussian kernel. In this case, the scale-space is linear, and the filter in (14) is just defined by convolution. The Gaussian kernel is one of the most studied in the theory of scale-spaces. It has some very interesting properties, one of them being that the signal Ψ obtained from it, is the solution of the heat equation (with Ψ_0 as initial condition) given by

$$\frac{\partial \Psi}{\partial t} = \Delta \Psi.$$

(For more details about the Gaussian scale-space, see the aforementioned references.)

One of the key facts that can be gleaned from the Gaussian example, is that the scale-space can be obtained as the solution of a partial differential equation called an *evolution equation*. This idea was developed in different works for evolution equations different from the classical heat equation. In what follows, we will describe scale-spaces for planar curves which are obtained as solutions of nonlinear evolution equations.

The second fundamental concept which we would like to emphasise is that of *invariant descriptor*. An invariant descriptor is a property of an object, which does not change when the object undergoes certain transformations. More precisely, a quantity Q is called an *invariant* of a Lie group $\mathcal{G}$ if whenever Q transforms into $\tilde{Q}$ by any transformation $g \in \mathcal{G}$, we obtain $\tilde{Q} = \Theta Q$, where Θ is a function of g alone. If $\Theta = 1$ for all $g \in \mathcal{G}$, Q is called an *absolute invariant*. We should add that the development of invariants of viewing transformations (Euclidean, similarity, affine, and projective maps) has received extended attention from the computer vision community in the last years. The topic is very important in areas such as object recognition. A special class of invariants is given by the *differential* invariants which are based only on local information, and are useful for the representation and recognition of objects under partial occlusions. An example of this kind of invariant is the Euclidean curvature. In our work, we have been especially interested in affine differential invariants.

The third component of our work comes from the theory of plane curve evolution. Here the curve regarded as the boundary of a planar domain, deforms in time. This deformation is governed by a partial, usually nonlinear, evolution equation. Different evolution equations can model different physical phenomena, such as crystal growth, the Huygens principle, and curve shortening processes. The theory has been well studied in areas such as computational physics, differential geometry, numerical analysis, and parabolic equations. We have introduced these ideas into computer vision in [49, 50, 51].

Motivated by the importance of affine transformations in computer vision, a new theory of affine curve evolution was recently developed [60, 61, 62, 63]. This theory, based on affine differential geometry and the theory of parabolic evolution equations, constitutes the basis of the work on multi-scale representations of signals and image processing in general. This is based in turn on a new nonlinear evolution diffusion equation which admits affine invariant solutions. The solution of this evolution equation determines an *affine* invariant scale-space for planar curves. Efficient computer implementation of this theory is possible due to a recently developed numerical algorithm for curve and surface evolution. In our continuing ARO research, we will be employing these algorithms to a number of problems in active vision, and visual control.

6 Bibliography

References

[1] J. Ball, C. Foias, J. W. Helton, and A. Tannenbaum, "On a local nonlinear commutant lifting theorem," *Indiana J. Mathematics* 36 (1987), pp. 693–709.

[2] J. Ball, C. Foias, J. W. Helton, and A. Tannenbaum, "Nonlinear interpolation theory in H^∞," in *Modelling, Robustness, and Sensitivity in Control Systems* (edited by Ruth Curtain), NATO-ASI Series, Springer-Verlag, New York, 1987.

[3] J. Ball, C. Foias, J. W. Helton, and A. Tannenbaum, "A Poincaré-Dulac approach to a nonlinear Beurling-Lax-Halmos theorem," *Journal of Math. Anal. and Applications* 139 (1989), pp. 496–514.

[4] J. Ball and J. W. Helton, "H^∞ control for nonlinear plants: connections with differential games," *Proc. of 28th Conference on Decision and Control*, Tampa, Florida, December 1989, pp. 956–962.

[5] J. Ball and J. W. Helton, "Nonlinear H^∞ control theory for stable plants," *MCSS* 5 (1992), pp. 233–261.

[6] B. Bamieh and J. Pearson, "A general framework for linear periodic systems with application to H^∞-sampled-data control," *IEEE Trans. Aut. Control* 37 (1992), 418–435.

[7] B. Bamieh, B. Francis, J. Pearson, and A. Tannenbaum, "A lifting technique for linear periodic systems with applications to sampled-data control," *Systems and Control Letters* 17 (1991), pp. 79–88.

[8] H. Bercovici, J. Cockburn, C. Foias, and A. Tannenbaum, "On structured tangential interpolation in robust control," *Proc. of 32nd IEEE Conference on Decision and Control*, December 1993.

[9] H. Bercovici, C. Foias, P. Khargonekar, and A. Tannenbaum, "A lifting technique for the robust stability analysis of systems with structured time-varying perturbations," *Proc. of Conference on Information Sciences and Systems*, Johns Hopkins University, March 1993.

[10] H. Bercovici, C. Foias, P. Khargonekar, and A. Tannenbaum, "On a lifting theorem for the structured singular value," to appear in *Journal of Math. Analysis and Applications*.

[11] H. Bercovici, C. Foias, and A. Tannenbaum, "On spectral tangential Nevanlinna-Pick interpolation" *Journal of Math. Analysis and Applications* 155 (1991), pp. 156–175.

[12] H. Bercovici, C. Foias, and A. Tannenbaum, "On skew Toeplitz operators I," *Operator Theory: Advances and Applications* 32 (1988), pp. 21–43.

[13] H. Bercovici, C. Foias, and A. Tannenbaum, "On the optimal solutions in spectral commutant lifting theory," *Journal of Functional Analysis* 101 (1991), pp. 38–49.

[14] H. Bercovici, C. Foias, and A. Tannenbaum, "A spectral commutant lifting theorem," *Trans. AMS* 325 (1991), pp. 741–763.

[15] H. Bercovici, C. Foias, and A. Tannenbaum, "Structured interpolation theory," *Operator Theory: Advances and Applications* 47 (1990), pp. 195–220.

[16] H. Bercovici, C. Foias, and A. Tannenbaum, "On a relative Toeplitz-Hausdorff theorem," to appear in *Operator Theory: Advances and Applications*.

[17] H. Bercovici and A. Tannenbaum, "The invariant subspaces of a uniform Jordan operator," *Journal Math. Analysis and Applications* 156 (1991), pp. 220–230.

[18] D. Bugajski, D. Enns, and A. Tannenbaum "Preliminary μ-synthesis design for the ATB-1000," *Proceedings of Tenth Army Conference on Applied Mathematics and Computing*, West Point, New York, 1992.

[19] D. Bugajski, D. Enns, and A. Tannenbaum, "Synthesis methods for robust nonlinear control," *Proceedings of ACC*, 1993.

[20] J. Cockburn, Y. Sidar, and A. Tannenbaum, "A constructive solution to the gain-phase margin problem," *Proc. of 31st IEEE Conference on Decision and Control*, Tucson, Arizona, 1992.

[21] R. F. Curtain, "H^∞-control for distributed parameter systems: a survey," *Proc. of 29th IEEE Conference on Decision and Control*, Honolulu, Hawaii, December 1990, pp. 22–26.

[22] J. C. Doyle, "Analysis of feedback systems with structured uncertainties," *IEE Proc.* 129 (1982), pp. 242–250.

[23] J. C. Doyle, B. Francis, and A. Tannenbaum, *Feedback Control Theory*, McMillan, New York, 1991.

[24] J. C. Doyle, K. Glover, P. Khargonekar, and B. Francis, "State space solutions to standard H^2 and H^∞ control problems," *IEEE Trans. Aut. Control* 34 (1989), pp. 831–847.

[25] D. Enns, C. Foias, T. Georgiou, M. Jackson, B. Schipper, and A. Tannenbaum, "On the nonlinear mixed sensitivity problem," *Proc. of 28th IEEE Conference on Decision and Control*, Tampa, Florida, December 1989, pp. 2673–2678.

[26] D. Enns, H. Özbay, and A. Tannenbaum, "Abstract model and control design for an unstable aircraft," *AIAA Journal of Guidance, Control, and Navigation* 15 (1992), pp. 498–508.

[27] C. Foias and A. Frazho, "Commutant lifting and simultaneous H^∞ and L^2 suboptimization," *SIAM J. Math. Anal.* 23 (1992), pp. 984–994.

[28] C. Foias, A. Frazho, and A. Tannenbaum, "On certain minimal entropy extensions appearing in dilation theory," *Linear Algebra and Its Applications* 137 (1990), pp. 213–238.

[29] C. Foias, A. Frazho, and A. Tannenbaum, "On combined H^∞–H^2 suboptimal interpolants," to appear in *Linear Algebra and Its Applications*.

[30] C. Foias, C. Gu, and A. Tannenbaum, "Nonlinear H^∞ optimization: a causal power series approach," to appear in in *SIAM J. Control and Optimization*.

[31] C. Foias, C. Gu, and A. Tannenbaum, "Intertwining dilations, intertwining extensions, and causality," *Acta Sci. Math. (Szeged)* 56 (1993), pp. 101–123.

[32] C. Foias, C. Gu, and A. Tannenbaum, "On the nonlinear standard H^∞ problem," submitted for publication in *Int. J. on Nonlinear and Robust control*.

[33] C. Foias, C. Gu, and A. Tannenbaum "On a causal linear optimization theorem," to appear in *Journal of Math. Analysis and Applications*.

[34] C. Foias and A. Tannenbaum, "On the parametrization of the suboptimal solutions in generalized interpolation," *Linear Algebra and its Applications* 124 (1989), pp. 145–164.

[35] C. Foias and A. Tannenbaum, "On the four block problem, I," *Operator Theory: Advances and Applications* 32 (1988), pp. 93–112.

[36] C. Foias and A. Tannenbaum, "On the four block problem, II : the singular system," *Operator Theory and Integral Equations* 11 (1988), pp. 726–767.

[37] C. Foias and A. Tannenbaum, "On the parametrization of the suboptimal solutions in generalized interpolation," *Linear Algebra and its Applications* 124 (1989), pp. 145–164.

[38] C. Foias and A. Tannenbaum, "Iterated commutant lifting for systems with rational symbol," *Operator Theory: Advances and Applications* 41 (1989), pp. 255–277.

[39] C. Foias and A. Tannenbaum, "Weighted optimization theory for nonlinear systems," *SIAM J. on Control and Optimization* 27 (1989), pp. 842–860.

[40] C. Foias and A. Tannenbaum, "Nonlinear H^∞ theory," in *Robust Control of Nonlinear Systems and Nonlinear Control*, edited by M. Kaashoek, J. van Schuppen, A. Ran, Birkhauser, Boston, 1990, pp. 267–276.

[41] C. Foias and A. Tannenbaum, "Causality in commutant lifting theory," *Journal of Functional Analysis* 118 (1993), pp. 407–441.

[42] C. Foias and A. Tannenbaum, "Global sensitivity reduction for saturated systems," in preparation.

[43] C. Foias , A. Tannenbaum, and G. Zames, "On the H^∞ optimal sensitivity problem for systems with delays," *SIAM J. Control and Optimization* 25 (1987), pp. 686–706.

[44] C. Foias, A. Tannenbaum, and G. Zames, "Some explicit formulae for the singular values of a certain Hankel operators with factorizable symbol," *SIAM J. Math. Analysis* 19 (1988), pp. 1081–1091.

[45] V. Kaftal, D. Larson, and G. Weiss, "Quasitriangular subalgebras of semifinite von Neumann algebras are closed," *J. Functional Analysis* 107 (1992), pp. 387–401.

[46] P. Khargonekar, K. Poolla, and A. Tannenbaum, "Robust control of linear time-invariant plants using periodic compensation," *IEEE Trans. Aut. Control* AC-30 (1985), pp. 1088–1098.

[47] P. Khargonekar and M. A. Rotea, "Mixed H^2/H^∞ control: a convex optimization approach," *IEEE Trans. Aut. Control* 36 (1991), pp. 14–24.

[48] P. Khargonekar and A. Tannenbaum, "Noneuclidean metrics and the robust stabilization of systems with parameter uncertainty," *IEEE Trans. Aut. Control* AC-30 (1985), pp. 1005–10013.

[49] B. B. Kimia, A. Tannenbaum, and S. W. Zucker, "Toward a computational theory of shape: An overview," *Lecture Notes in Computer Science* 427, pp. 402-407, Springer-Verlag, New York, 1990.

[50] B. B. Kimia, A. Tannenbaum, and S. W. Zucker, "Shapes, shocks, and deformations, I," to appear in *International Journal of Computer Vision.*

[51] B. B. Kimia, A. Tannenbaum, and S. W. Zucker, "On the evolution of curves via a function of curvature, I: the classical case," *J. of Math. Analysis and Applications* 163, pp. 438-458, 1992.

[52] K. Lenz, H. Özbay, A. Tannenbaum, J. Turi, and B. Morton, "Frequency domain analysis and robust control design for an ideal flexible beam," *Automatica* 27 (1991), pp. 947–961.

[53] H. Özbay, M. C. Smith and A. Tannenbaum, "Controller design for unstable distributed plants," *Proc. of ACC*, 1990, pp. 1583–1588.

[54] H. Özbay, M. C. Smith and A. Tannenbaum, "Mixed sensitivity optimization for unstable infinite dimensional systems," *Linear Algebra and Its Applications* 178 (1993), pp. 43–83.

[55] H. Özbay and A. Tannenbaum, "A skew Toeplitz approach to the H^∞ control of multivariable distributed systems," *SIAM J. Control and Optimization* 28 (1990), pp. 653–670.

[56] H. Özbay and A. Tannenbaum, "On the synthesis of H^∞ optimal controllers for infinite dimensional plants," in *New Trends and Applications in Distributed Parameter Control Systems,* edited by G. Chen, E. B. Lee, W. Littman, L. Marcus, Marcel Dekker, New York, 1990, pp. 271–301.

[57] H. Özbay and A. Tannenbaum, "On the structure of suboptimal H^∞ controllers in the sensitivity minimization problem for distributed stable plants," *Automatica* 27 (1991), pp. 293–305.

[58] G. Pisier, "Interpolation between H^p spaces and non-commutative generalizations, I" *Pacific Journal of Mathematics* 155 (1992), pp. 341–368.

[59] M. G. Safonov, *Stability Robustness of Multivariable Feedback Systems,* MIT Press, Cambridge, Mass., 1980.

[60] G. Sapiro and A. Tannenbaum, "On affine plane curve evolution," *Journal of Functional Analysis* 119 (1994), pp. 79–120.

[61] G. Sapiro and A. Tannenbaum, "Affine invariant scale-space," *International Journal of Computer Vision* 11 (1993), pp. 25–44.

[62] G. Sapiro and A. Tannenbaum, "Area and length preserving geometric invariant scale-spaces," to appear in *IEEE Pattern Analysis and Machine Intelligence.*

[63] G. Sapiro and A. Tannenbaum, "Invariant curve evolution and image analysis," *Indiana University J. of Mathematics* 42 (1993), pp. 985–1009.

[64] G. Sapiro and A. Tannenbaum, "Image smoothing based on an affine invariant flow," *Proceedings of the Conference on Information Sciences and Systems,* Johns Hopkins University, March 1993.

[65] J. Shamma, "Robust stability with time-varying structured uncertainty," to appear in *IEEE Trans. Aut. Control.*

[66] A. Sideris and M. Safonov, "A design algorithm for the robust synthesis of SISO feedback control systems using conformal maps and H^∞ theory," *Proc. of ACC*, 1984, pp. 1710–1715.

[67] A. Tannenbaum, "On the multivariable gain margin problem," *Automatica* 22 (1986), pp. 381–384.

[68] A. P. Witkin, "Scale-space filtering," *Int. Joint. Conf. Artificial Intelligence*, pp. 1019-1021, 1983.

7   Papers of Allen Tannenbaum and Collaborators under DAAL-03-91-G-0019

(All the authors are listed in alphabetical order on the following papers.)

Journal Papers and Book Chapters

1. "Frequency domain analysis and robust control design for an ideal flexible beam," (with V Iens, B. Morton, H. Özbay, J. Turi), *Automatica* 27 (1991), 947–961.

2. "A spectral commutant lifting theorem" (with H. Bercovici and C. Foias), *Transactions* S 325 (1991), 741–763.

3. "On spectral tangential Nevanlinna-Pick interpolation" (with H. Bercovici and C. Foias), *Journal of Math. Analysis and Applications* 155 (1991), 156–176.

4. "The invariant subspaces of a uniform Jordan operator" (with H. Bercovici), *Journal Math. Analysis and Applications* 156 (1991), 220–230.

5. "On the evolution of curves via a function of curvature, I: the classical case" (with B. Kimia and S. Zucker), *Journal of Math. Analysis and Applications* 163 (1992), 438–458.

6. "On the optimal solutions in spectral commutant lifting theory" (with H. Bercovici and C. Foias), *Journal of Functional Analysis* 101 (1991), 38–49.

7. "Abstract model and control design for an unstable aircraft" (with Dale Enns and Hitay Özbay), *AIAA Journal of Guidance, Control, and Navigation* 15 (1992), 498–508.

8. "A lifting technique for linear periodic systems with applications to sampled-data control" (with B. Bamieh, B. Francis, J. B. Pearson), *Systems and Control Letters* 17 (1991), 79–89.

9. "On the structure of suboptimal H^∞ controllers in the sensitivity minimisation problem for distributed stable plants" (with Hitay Özbay), *Automatica* 27 (1991), 293–305.

10. "Mixed sensitivity optimisation for a class of unstable infinite dimensional systems" (with H. Özbay and M. Smith), *Linear Algebra and Its Applications* 178 (1993), 43–83.

11. "Intertwining dilations, intertwining extensions, and causality" (with C. Foias and C. Gu), *Acta Sci. Math.* 56 (1993), 101–123.

12. "On combined H^∞-H^2 suboptimal interpolants" (with C. Foias and A. Frasho), to appear in *Linear Algebra and Its Applications*.

13. "Some mathematical problems in computer vision" (with A. Bruckstein), *Acta Math. Appl.* 30 (1993), 125–158.

14. "Shapes, shocks, and deformations, I: the components of shape and the reaction-diffusion space" (with B. Kimia and S. Zucker), to appear in *International Journal of Computer Vision*.

15. "On a causal linear optimisation theorem" (with C. Foias and C. Gu), to appear in *Journal of Math. Analysis and Applications*.

16. "On affine plane curve evolution" (with G. Sapiro), *Journal of Functional Analysis* 119 (1994), 79–120.

17. "Affine invariant scale-space" (with G. Sapiro), *International Journal of Computer Vision* 11 (1993), 25–44.

18. "Causality in commutant lifting theory" (with C. Foias), *Journal of Functonal Analysis* 118 (1993), 407–441.

19. "On invariant curve evolution and image analysis" (with G. Sapiro), *Indiana Univ. Journal of Mathematics* 42 (1993), 985–1009.

20. "Nonlinear H^∞ optimisation: a causal power series approach" (with C. Foias and C. Gu), to appear in *SIAM J. Control and Optimization.*

21. G. Sapiro and A. Tannenbaum, "Area and length preserving geometric invariant scale-sapces," to appear in *IEEE Trans. on Pattern Analysis and Machine Intelligence.*

22. "On a lifting theorem for the structured singular value" (with H. Bercovici, C. Foias, and P. Khargonekar), to appear in *Journal of Math. Analysis and Applications.*

23. "A relative Toeplitz-Hausdorff theorem" (with H. Bercovici and C. Foias), to appear in in *Operator Theory: Advances and Appications.*

24. "Continuity of the spectrum on closed similarity orbits" (with H. Bercovici and C. Foias), *Integral Equations and Operator Theory* 18 (1994), 242–246.

25. "Differential invariant signatures and flows in computer vision: a symmetry group approach" (with P. Olver and G. Sapiro), to appear as a book chapter in *Geometric Diffusion*, edited by Bart Romeny.

26. "Classification and uniqueness of invariant geometric flows" (with P. Olver and G. Sapiro), to appear in *Comptes Rendus.*

27. "On optimal control methods in computer vision and image processing" (with B. Kimia and S. Zucker), to appear as a book chapter in *Geometric Diffusion*, edited by Bart Romeny.

29. "On the nonlinear standard H^∞ problem" (with C. Foias and C. Gu), submitted for publication in *Int. J. Nonlinear and Robust Control.*

30. "Invariant theory and families of systems," *Mathematical Systems Theory: The Influence of Rudolf Kalman*, edited by A. C. Antoulas, Springer-Verlag, 1991, 327-345.

31. "Entropy scale-space" (with B. Kimia and S. Zucker), in *Visual Form*, edited by C. Arcelli, pages 333-344, Plenum Press, New York, 1992.

32. "Frequency domain methods for the H^∞ optimisation of distributed systems," in *Analysis and Optimization of Systems: State and Frequency Domain Approaches for Infinite Dimensional Systems*, edited by Ruth Curtain, Lecture Notes in Control and Information Sciences 185, Springer-Verlag, New York, 1992.

33. "Generalised interpolation theory and its application to robust system design," *Digital and Numeric Techniques and Their Applications in Control Systems*, edited by C. T. Leondes, Academic Press, 1993, pages 163-217.

34. "From curve detection to shape description" (with A. Dobbins, L. Iverson, B. Kimia, and S. Zucker), in *Computer Vision: Systems, Theory, and Applications*, edited by A. Basu and X. Li, World Scientific, Singapore, 1993, pages 25-39.

35. "Stability margin optimisation via interpolation and conformal mappings" (with J. Cockburn and Y. Sidar), submitted for publication in *IEEE Trans. Aut. Control.*

Refereed Conference Publications

36. "H^∞ optimal controllers for as distributed model of an aircraft" (with D. Enns and H. Özbay), *Proceedings of 30th IEEE Conference on Decision and Control*, Brighton, England, 1991.

37. "Computational methods for the H^∞ control of distributed systems," *Proceedings of 30th IEEE Conference on Decision and Control*, Brighton, England, 1991.

38. "On the optimal two block H^∞ compensators for distributed unstable plants" (with H. Özbay), *Proceedings of ACC*, 1992.

39. "Conservation laws and the evolution of shape" (with B. Kimia and S. Zucker), *Human/Machine Vision Workshop, SPIE Meetings*, Falmouth, MA, 1991.

40. "A conservation-minded approach to shape" (with B. Kimia and S. Zucker), *Proceedings of IEEE Conference on Intelligent Control*, Philadelphia, 1991.

41. "A constructive solution to the gain-phase margin problem" (with Juan Cockburn and Yariv Sidar), *Proceedings of 31st IEEE Conference on Decision and Control*, Arizona, 1992.

42. "Synthesis methods for robust nonlinear control" (with D. Bugajski and Dale Enns), *Proceedings of ACC*, 1993.

43. "Preliminary mu-synthesis design for the ATB-1000" (with D. Bugajski and Dale Enns), *Proceedings of Tenth Army Conference on Applied Mathematics and Computing*, West Point, New York, 1992.

44. "Exploring the shape manifold: the role of conversation laws" (with B. Kimia and S. Zucker), *Proceedings of the Shape in Picture NATO Conference*, Driebergen, Netherlands, September, 1992.

45. "Causal commutant lifting in $H^2(D^n)$ and its applications in nonlinear H^∞ control" (with C. Foias and C. Gu), *Proceedings of Workshop on Robust Control*, IMA, Minneapolis, Minnesota, September 1992.

46. "Non-linear shape approximation via the entropy scale space" (with B. Kimia and S. Zucker), *Proceedings of the SPIE Geometric Methods of Computer Vision Conference*, San Diego, 1993.

47. "Formulating invariant heat-type curve flows" (with G. Sapiro), *Proceedings of the SPIE Geometric Methods of Computer Vision Conference*, San Diego, 1993.

48. "A lifting technique for the robust stability analysis of systems with structured time-varying perturbations" (with H. Bercovici, C. Foias, and P. Khargonekar), *Proceedings of the Conference on Information Sciences and Systems*, Johns Hopkins University, March 1993.

49. "Image smoothing based on an affine invariant flow" (with G. Sapiro), *Proceedings of the Conference on Information Sciences and Systems*, Johns Hopkins University, March 1993.

50. "On structured tangential interpolation in robust control" (with H. Bercovici, J. Cockburn, and C. Foias), *Proceedings of 32nd IEEE Conference on Decision and Control*, December 1993.

51. "Pseudorational functions and H^∞ theory" (with Y. Yamamoto), to appear in *Proceedings of ACC*, 1994.

52. "Skew Toeplitz theory and pseudorational transfer functions" (with Y. Yamamoto), submitted to 1994 IEEE CDC.

53. "Experiments on geometric image enhancement" (with M. Kaveh, G. Sapiro, Y. L. You), submitted to First IEEE International Conference on Image Processing, Austin, 1994

Books

1. J. Doyle, B. Francis, and A. Tannenbaum, *Feedback Control Theory*, MacMillan, New York, 1991.

2. C. Foias, H. Özbay, and A. Tannenbaum, *H^∞ Control of Distributed Parameter Systems*, in preparation, 1994. First draft is available.

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